PHASE SHIFT CALCULATIONS IN AXIALLY-MAGNETIZED FERRITES USING THE FINITE ELEMENT METHOD

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ABSTRACT

A vector finite element solver is used to calculate the phase shift in uniform axially-magnetized gyromagnetic waveguides. Ferrite materials are characterized in the solver using standard material data, the applied field and the frequency. Calculations are compared with experimental results for two typical waveguide cross-sections: a quadrupally-ridged Faraday Rotation section and a reciprocal Reggia-Spencer section.

INTRODUCTION

It is well understood that waveguides containing axially magnetized ferrite materials are often used as phase shifters at microwave frequencies [1]. A classic non-reciprocal arrangement is based on the Faraday rotation principle. These type of phase shifters are constructed using waveguide cross-sections which support degenerate, orthogonal, linearly-polarized modes. Analysis and design of both Faraday rotation phase shifters and related dual-mode phase shifters is described in the literature [1,9]. In waveguides with less than four-fold symmetry, a suppressed-rotation mechanism has been described and employed to obtain reciprocal phase shifters [5,6]. A CAD approach to the specification of waveguides for phase shifters requires a numerical method which calculates the propagation constant at the required frequency. A vector-field analysis is necessary to model the hybrid modes which occur in waveguides when the tensor permeability used to characterize the ferrite media is introduced. In addition, for the modelling of practical geometries it is preferable to specify the ferrite characteristics in terms of the material parameters and the applied magnetic field instead of the tensor permeability entries (κ, μ) commonly used in analytical solutions of gyromagnetic waveguides [1].

The Finite Element (FE) Method is well suited to the analysis of waveguides with arbitrary geometry. A FE formulation in terms of the vector magnetic field is used here to calculate the propagation constant and modal fields at a specified frequency. Spurious modes are avoided through the use of covariant-projection elements [8]. The formulation results in a generalized eigenvalue matrix problem where the propagation constant is the eigenvalue. To validate this FE formulation, axi-symmetric geometries with known solutions were recently analysed and the results are described elsewhere [2]. In this paper the application to more practical waveguide geometries such as those used in phase shifters are considered. Two aspects of the original formulation have been extended and will be described: data input in terms of experimental parameters instead of the components of the tensor permeability; identification of symmetry planes and specification of appropriate boundary conditions. Results will be presented for two different sorts of phase shifters; Faraday rotation sections and Reggia-Spencer sections.

FINITE ELEMENT FORMULATION

The time-harmonic magnetic field within a waveguide containing gyromagnetic media satisfies the vector wave equation

$$\nabla \times (\epsilon_f^{-1} \nabla \times \mathbf{H}) - k_0^2 \hat{\mu}_r \mathbf{H} = 0 \qquad (1)$$

where k_0 is the wavenumber of free space. It is assumed here that the waveguide is uniform in the z direction, sop all field components have a z-dependence of $e^{-j\beta z}$ where β is the phase constant. To evaluate the phase constants and the modal field patterns at a specified frequency, the stationary point of the following functional is found

$$F(\mathbf{H}') = \int_S \Big\{ \epsilon_f^{-1} |\nabla_t \times \mathbf{H_t}'|^2 - k_0^2 \mathbf{H_t}'^* \cdot \hat{\mu}_{tt} \mathbf{H_t}' + \beta^2 \left[\epsilon_f^{-1} |\nabla_t H_z' + \mathbf{H_t}'|^2 - k_0^2 H_z'^* \mu_{zz} H_z' \right] \Big\} dS \qquad (2)$$

In this equation both the magnetic field and tensor permeabilty have been split into transverse and axial parts. The magnetic fields $\mathbf{H}'$ are transformations of the magnetic field $\mathbf{H}$ described in [4,5]. The finite element solution of (2) is obtained by substituting the trial functions of the meshed region into the functional equation. The first variation of the functional is then set to zero and the solution is calculated from the resulting generalized eigenvalue matrix equation of the form

$$[A]\overline{H} = \lambda^2[B]\overline{H} \tag{3}$$

where, in general, the $n \times n$ matrices [A] and [B] are Hermitian and $\overline{H}$ is an n component vector of unknown field coefficients. The eigenvalue λ^2 corresponds to the phase constant β^2.

FERRITE CHARACTERISTICS

Phase constant calculations in ferrite waveguides are often presented in terms of a κ/μ parameter derived from the permeability tensor. This parameter is chosen for mathematical convenience. In general it is a function of the frequency, applied field and magnetisation and so it is not easy to relate the resulting characteristics to measured results. For waveguide design purposes, phase constant calculations in terms of either frequency or applied field are more useful.

In ferrites which are saturated the relationship between the tensor entries and the material characteristics is well-known [1]. The value of the magnetisation is known and for any applied field and frequency the tensor components can be calculated. Difficulties arise for partially magnetized ferrites where the value of the magnetisation for a particular applied field is not known. A set of formulae suitable for all magnetisations higher than the remanence level have been proposed by Hansson and Filipsson [3]. Assuming a tensor of the form

$$\hat{\mu}_r = \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \tag{4}$$

where the applied dc magnetic field is along the z-axis. The values of the tensor components are [3]

$$\begin{aligned} \mu_{zz} &= \mu_0^{1-p^{5/2}} \\ \mu &= \mu_0 + (1-\mu_0)p^{3/2} + \frac{\eta^2 H_0 M}{(\eta H_0)^2 - \omega^2} \\ \kappa &= \frac{w\eta M}{(\eta H_0)^2 - \omega^2} \end{aligned} \tag{5}$$

where ω is the frequency of operation, H_0 is the internal field and η is the gyromagnetic ratio. The variable p is the ratio of the magnetisation M to the saturation magnetisation M_s [3]

$$p = \frac{M}{M_s} = a_1 + (1-a_1)\left\{\coth(a_2 H_0) - \frac{1}{a_2 H_0}\right\}$$

where a_1 and a_2 are constants derived from the hystersis curve of the material: a_1 is equal to the ratio of the remanent and saturation magnetisations and a_2 is related to the slope of $dB/dH|_{H=0}$. In eqn. (4), μ_0 is the scalar permeability in the demagnetized state

$$\mu_0 = \frac{1}{3} + \frac{2}{3}\sqrt{1 - \frac{(\eta M)^2}{\omega^2}}$$

where the effects of the anisotropy field have been neglected. The values predicted by these formulae agree reasonably well with measured data for a range of ferrites [3]. These formulae have been incorporated into the FE solver so that the components of the tensor permeability are calculated from the material characteristics for any specified applied field and frequency.

SYMMETRY CONDITIONS

Vector finite element solutions of waveguide cross-sections is a computationally expensive procedure. Reductions in the size of the meshed region of the waveguide cross-section can lead to significant savings in computer time. In waveguide geometries with materials characterized by scalar material properties, mirror planes of symmetry are used to reduce the size of the meshed region. The mirror planes are modelled as electric or magnetic walls. Waveguides with axially magnetized ferrite media do not support mirror symmetry but may have rotational symmetry. These rotational planes of symmetry relate the vector field components along two azimuthal planes in the waveguide to each other

$$\mathbf{H}(r,\theta) = \mathbf{m} \cdot \mathbf{H}(r, \theta + \phi) \tag{6}$$

where ϕ is the angle of rotation and $\mathbf{m}$ is a 3×3 transformation matrix whose values are defined by the symmetry of the required modal solution. Periodic boundary conditions have been implemented in the finite element solver to take advantage of this rotational symmetry. These type of boundary conditions were used for the axi-symmetric geometries described in [2] and are employed here for more general geometries. Introducing these type of boundary conditions does not alter the symmetry of the matrix nor does it involve evaluating any line integrals over the mesh boundaries.

PHASE SHIFT CALCULATIONS

Faraday-rotation sections may be constructed using an axially magnetized ferrite rod centrally positioned in a circular waveguide. When the ferrite is magnetized, the circular polarized modes in the waveguide travel at different phase velocities. Over a length of uniform waveguide L the Faraday angle of rotation of the propagating linearly-polarized mode is $\phi = (\beta_+ - \beta_-)/2L$. For wide-band applications, a nearly constant rotation with frequency is required. One way of achieving this is to lower the cut-off frequency of the propagating mode, thereby reducing the effect of dispersion.

A typical quadrupally-ridged Faraday rotation cross-section is illustrated in fig. 1a. This type of geometry has been proposed by Chait and Sakiotis [4] among others, to improve the bandwidth characteristics of Faraday rotator sections. Phase shift calculations were obtained using a mesh of about 20 elements defined over a quarter section of the waveguide. To calculate the phase constant of the dominant $HE_{\pm 1,1}$ circularly-polarized modes, the relationship between the components in eqn. (6) with the matrix

$$\mathbf{m} = \begin{pmatrix} 0 & \pm i & 0 \\ \pm i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is used to specify the boundary condition. Each mesh was solved twice to obtain the positive and negative circularly-polarized modes. Figure 2 shows the calculated phase shift of the Faraday rotator section in fig. 1a, with and without ridges. There is a significant improvement in the bandwidth when the ridges are in place. Measured data for the improvement in a 90° rotator based on a similar geometry described in [4] is also shown in fig. 2 for comparison. The reduced phase shift in the measured results may be due to dielectric loading which was possibly used together with the ridges to enhance the bandwidth characteristics of the rotator. In addition, the ferrite rod is often tapered at both ends to improve matching. No allowance was made for this in the finite element calculations where a uniform rod was assumed.

Rectangular waveguides with a centrally positioned ferrite rod are used as variable phase shifters. This type of phase shifter, first proposed by Reggia and Spencer [5], is usually operated at low applied dc fields where the differential phase shift varies rapidly with applied dc field. One well-known model for the operation of these devices is a coupled mode approach based on the the modes in a similar waveguide with a ferrite slab [6]. Comparison between experimental results and coupled mode results from [6] and FE calculations are shown in fig. 3. The presence of the ferrite implies that a quarter section of the geometry is not sufficient to obtain a solution and half the geometry must be modelled. The vertical plane of symmetry in fig. 1b was used with a boundary condition specified by eqn. (6) with $\mathbf{m}$ equal to a unit matrix. Each section was solved twice at each frequency; once with zero applied field and next with a small applied field. Calculations for the variation in phase shift with applied field for different sized ferrites are shown in fig. 4. The dramatic change in phase shift for different-sized ferrites is typical of these class of phase shifters [5,6]. For low values the applied field, the FE results may not correlate exactly with experimental values because of the limitations of the model used to characterize the ferrite [3]. However the results in figs. 3 and 4 indicate that the FE solver models the behaviour of these partially-magnetized phase shifters reasonably well.

CONCLUSIONS

A three-component vector finite element solver has been used to calculate phase constants in waveguides containing axially-magnetized ferrite media. Ferrite characteristics are specified in terms of the applied field, frequency and material characteristics including saturation magnetisation and data from the hystersis curve. Periodic boundary conditions are used to reduce the size of the meshed region. Phase shifts have been calculated for two examples, a Reggia-Spencer phase shifter and a ridged Faraday rotation section and in each case comparisons are made with measured data in the literature.

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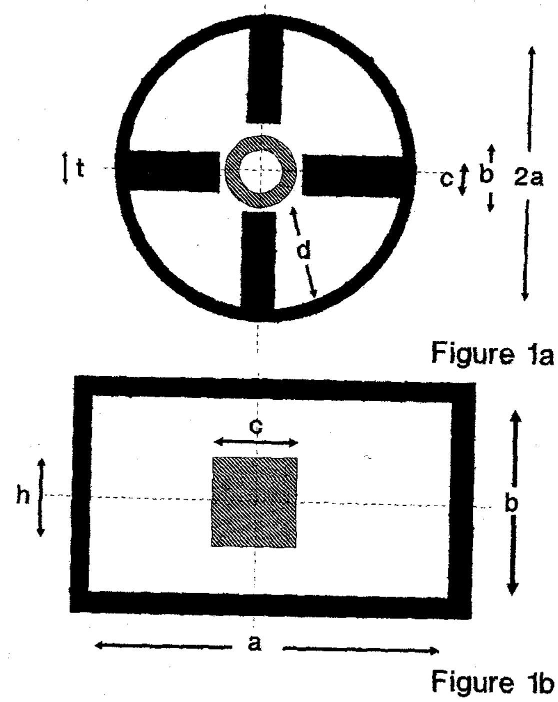

Fig. 1: Geometry of phase shifter cross-sections (1a):Quadrupally-ridged Faraday rotator: $a = 11.944$ mm, $b = 6.426$ mm, $c = t = 3.175$ mm. Waveguide is air-filled except for hat-ched region which is TT-390 ferrite: $M_s = 2150$G, $H_{dc} = 384$ Oe, and $\epsilon_f = 12.7$. (1b): Reggia-Spencer phase shifter: $b/a = 0.444$, $h/a = 0.222$. Waveguide is air-filled except for hatched region which is ferrite: $k_m a = 2.9$, $\epsilon_f = 13$.

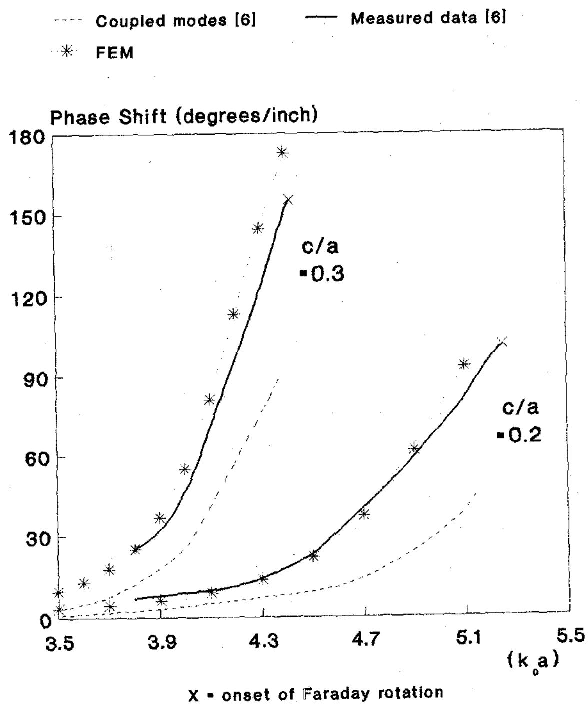

Fig. 3: Variation in differential phase shift with frequency. The coupled mode results were calculated for $b/h = 1$. The finite element method results were obtained using the geometry in fig. 1b with $H_{dc} = 0.0$ and $H_{dc} = 1$ Oe.

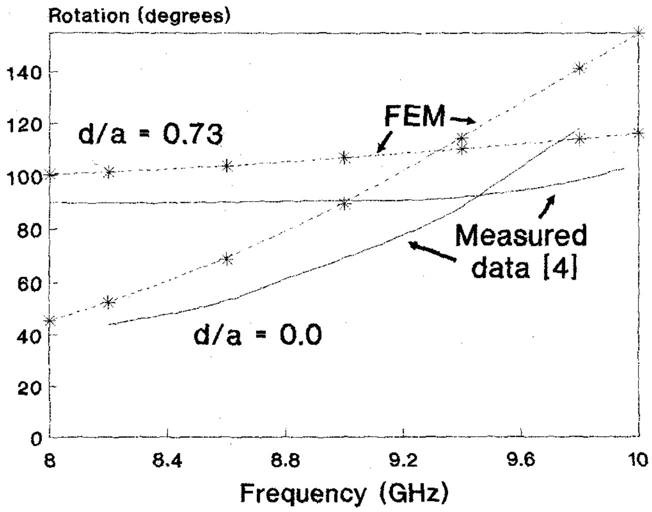

Fig. 2: Rotation over a Faraday Rotation section, with and without ridges. The length of the section is 50.8 mm.

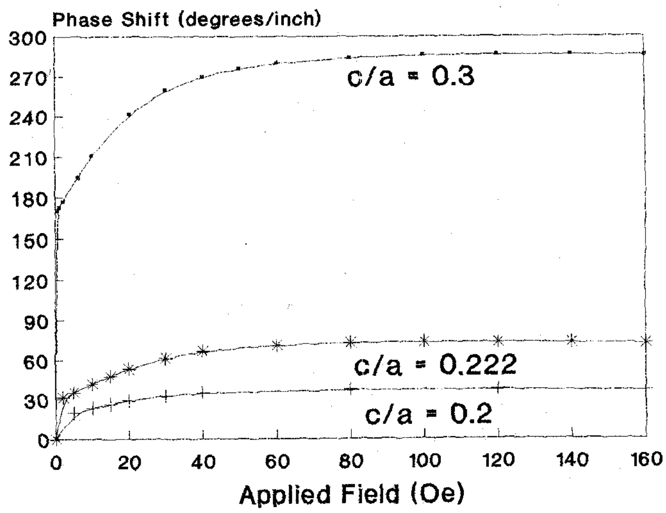

Fig. 4: Finite element calculations for the variation in differential phase shift with applied dc magnetic field for different ferrite widths. $k_0 a = 4.4$